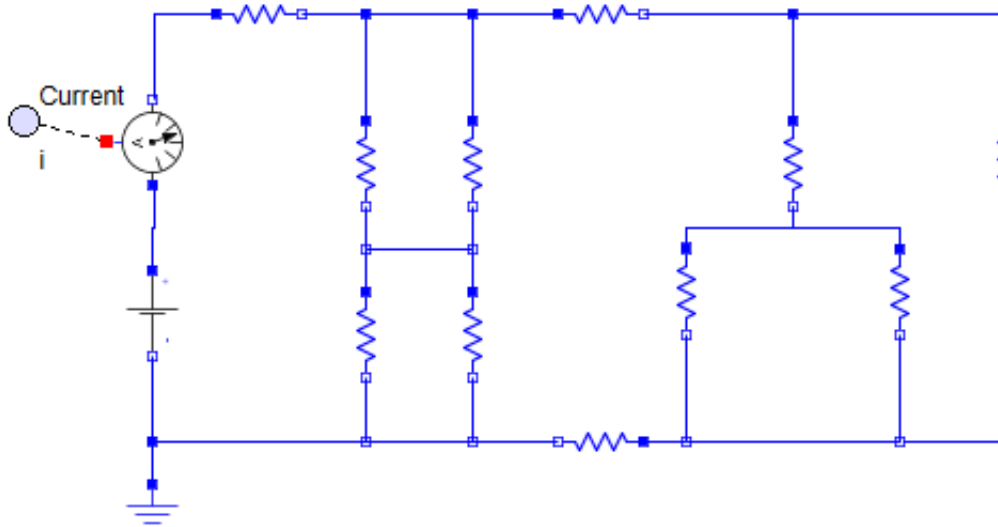# Series and Parallel Resistive Circuits

The configuration of circuit elements clearly affects the behaviour of a circuit. Resistors connected in series or in parallel are very common in a circuit and act as a voltage or current divider. We use Kirchhoff's laws to understand the extent of these configurations and to analyze resistive circuits. These principles also help introduce 'equivalent' circuits, exploring ways to simplify a circuit for easier analysis.

This module reviews Kirchhoff's current and voltage laws, and then explores resistive circuits using these principles. Analysis of resistive circuits for resistors in series and in parallel is explained, as well as introducing current and voltage division methods. Equivalent resistors and their respective equations are presented as well.

▼ Kirchhoff's Laws

An electric circuit consists of many elements connected to each other. The place where the element connections happen are called **nodes**. Multiple circuit elements can be connected to a node.
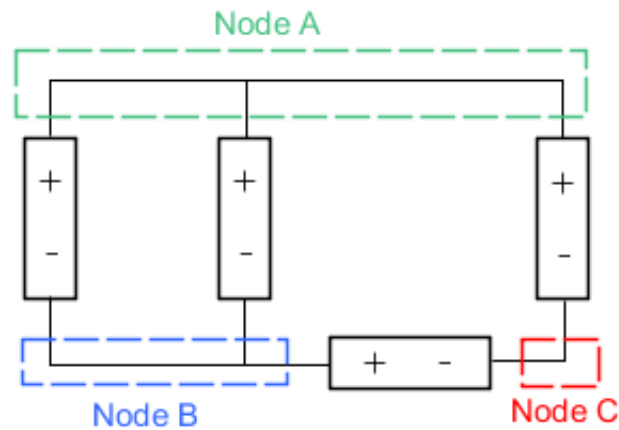
Figure 1: Nodes in a circuit with arbitrary elements

Circuits can be arranged in a wide variety of configurations. An equivalent circuit can take many configurations, but the circuit will behave the same way in each. With equivalent circuits, the number of elements and nodes should be consistent with the other circuit, and the elements should be connected to the corresponding nodes. Other forms of equivalent circuits may combine two or more circuit elements as well, as long as the circuit behaves identically.

Kirchhoff's current law (KCL) draws the relation for the interaction of current at a node. It states,

> The algebraic sum of all currents entering a node at any instant is zero.

This makes sense because there are no energy storing elements in a node. The direction of the currents are accounted for with positive or negative signs for exiting or entering currents. The polarities can be assumed either way for entering or exiting currents if the method remains consistent throughout the circuit analysis.

A loop is a closed path through a circuit which does not meet the same node more than once. Kirchhoff's voltage law (KVL) describes the principle for voltage in a loop.

> The algebraic sum of the voltages around any loop in a circuit is identically zero at any time.

Similar to KCL, the algebraic sum implies that the polarity of the voltage across each element will be included. We could assume that voltages added to the circuit, like from a voltage source, are given the negative convention while elements which reduce the voltage such as resistors will be given a positive value. This convention must remain consistent throughout the analysis of a circuit.

Kirchhoff's laws are used in the analysis of resistive circuits, shown in the sections below.

Series Resistors

A series circuit has the circuit elements connected to each other in a way such that it must be traversed one element to the next. Consider the circuit shown in fig. 2. A few resistors are connected in a circuit in series.
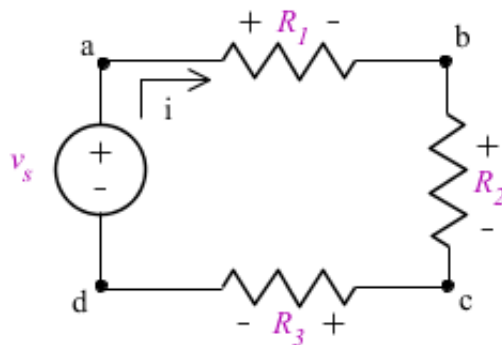

Figure 2: Circuit of resistors in series

In this series circuit, the current will be constant around the loop. By KCL, the current entering each node is equal to the current leaving each node (in this series circuit, each node has only one entering and exiting current). Also, the current does not change crossing a resistor.

The sum of voltages around the loop in fig. 2 must be equal to zero by KVL. Assume the voltages v_1, v_2, and v_3 for the resistors R_1, R_2, and R_3 respectively. The sum of the voltages around this loop would be

$$-v_s + v_1 + v_2 + v_3 = 0$$

Ohm's law shows that we can express these voltages with the resistors and the current (which is constant).

$$v_s = R_1 \cdot i + R_2 \cdot i + R_3 \cdot i$$

Rearrange this to isolate the current through this series circuit.

$$i = \frac{v_s}{R_1 + R_2 + R_3}$$

We can find the voltage across the nth resistor. Using ohm's law, the current through the nth resistor is equal to the voltage across the resistor divided by the resistance. Substitute this in for i and multiply by the resistance.

$$v_n = \frac{R_n \cdot v_s}{R_1 + R_2 + R_3}$$

A more general form describes the voltage across a resistor R_n for a set of N resistors connected in series as shown in fig. 3. This is the voltage divider method.

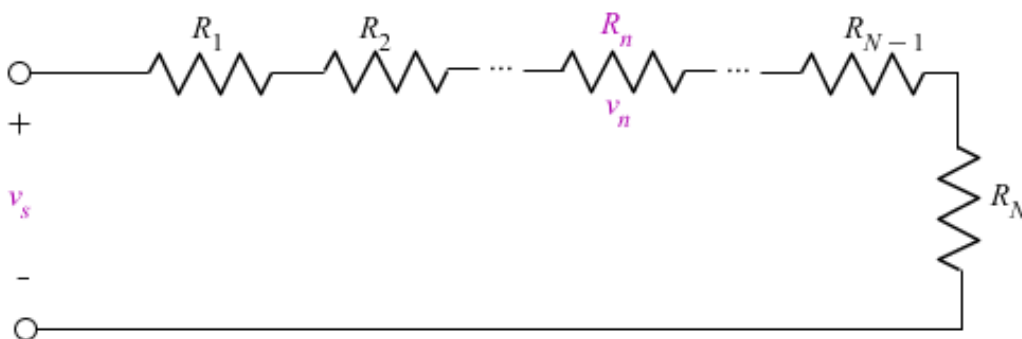$$v_n = \frac{R_n}{R_1 + R_2 + R_3 + ... + R_N} \cdot v_s$$

Figure 3: General required form to use voltage division

A series of resistors can be reduced to make a circuit simpler. In figure 3 we see resistors connected in series. The voltage across the set of resistors is known and can also be

represented as the sum of the voltages across each resistor.

$$v_s = v_1 + v_2 + v_3 + \dots + v_N$$

... Eq. (6)

Using Ohm's law, replace these voltages. The current will be constant throughout the series of resistors.

$$v_s = i \cdot R_1 + i \cdot R_2 + i \cdot R_3 + \dots + i \cdot R_N$$

... Eq. (7)

Factor the current out. By dividing the left side by the current, ohm's law reveals the equivalent resistance.

$$R_s = R_1 + R_2 + R_3 + \dots + R_N$$

... Eq. (8)

The equivalent resistance of resistors connected in series is the sum of the resistances.

Parallel Resistors

A parallel connection in a circuit means that 2 or more elements are connected to the same nodes at both connections. Figure 4 below illustrates this.
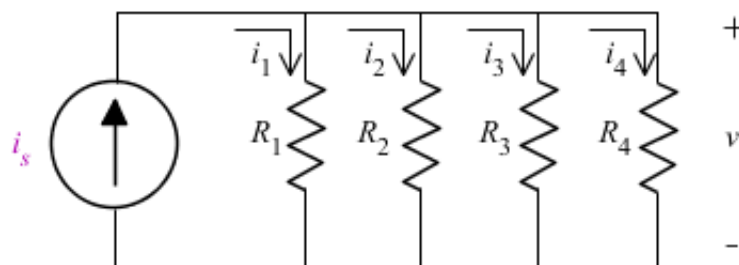

Figure 4: Parallel circuit of resistors

The voltage v across all of the resistors is constant in this circuit. The current source produces a current which splits up at the node. Using Kirchhoff's current law, we know that the algebraic sum of all currents entering a node will be zero. The sum of the currents leaving the node (across each resistor) will be equal to the current supplied by the source.

$$-i_s + i_1 + i_2 + i_3 + i_4 = 0$$

... Eq. (9)

From Ohm's law,

$$i_s = \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3} + \frac{v}{R_4}$$

... Eq. (10)

Factoring and dividing by the constant voltage, the equivalent resistance can be found for these resistors.

$$\frac{i_s}{v} = \frac{1}{R_s} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

... Eq. (11)

Of course, the more general form includes a set of N resistors connected in parallel with the same voltage across all of the resistors.

$$\frac{1}{R_s} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_N}$$

... Eq. (12)

We can find the current through each of the resistors with current division. Starting from Eq. (10) in the general form,

$$i_s = \left(\frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_N} \right) \cdot v$$

... Eq. (13)

The voltage across each resistor is the same. Use Ohm's law with the voltage across the nth resistor of interest, R_n.

$$i_s = \left(\frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_N} \right) \cdot i_n \cdot R_n$$

... Eq. (14)

Rearrange this equation and solve for i_n.

$$i_n = \frac{i_s}{R_n} \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_N} \right)^{-1}$$

... Eq. (15)

This is known as current division. This can also be understood as equating the voltage across the R_n branch to the voltage across the equivalent resistance R_s because the voltage across a parallel connection is the same.

$$i_n \cdot R_n = i_s \cdot R_s$$

Examples with MapleSim

Example 1: Equivalent Resistors

Problem Statement: Determine the current across the designated resistor in this circuit.
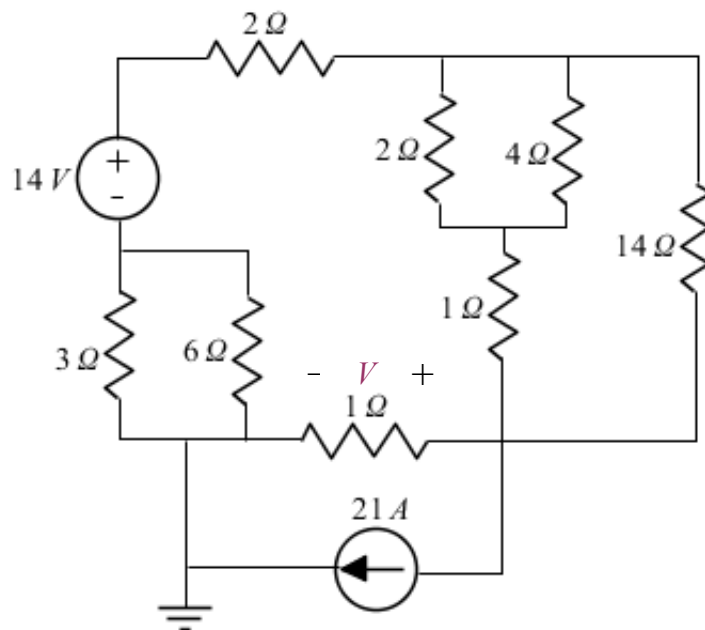

Figure 5: Resistive circuit with 2 sources

Analytical Solution

restart;

Data:

$Vs := 14:$

$Is := 21:$

$R1 := 2:$

$R2 := 2:$

$R3 := 4 :$
$R4 := 1 :$
$R5 := 14 :$
$R6 := 1 :$
$R7 := 3 :$
$R8 := 6 :$

Solution:

Begin by simplifying the circuit as much as possible before analyzing it. Combine the parallel resistors.

$$R_{eq1} := \left(\frac{1}{R2} + \frac{1}{R3} \right)^{-1} = \frac{4}{3}$$

$$R_{eq2} := \left(\frac{1}{R7} + \frac{1}{R8} \right)^{-1} = 2$$

The circuit now looks like this.
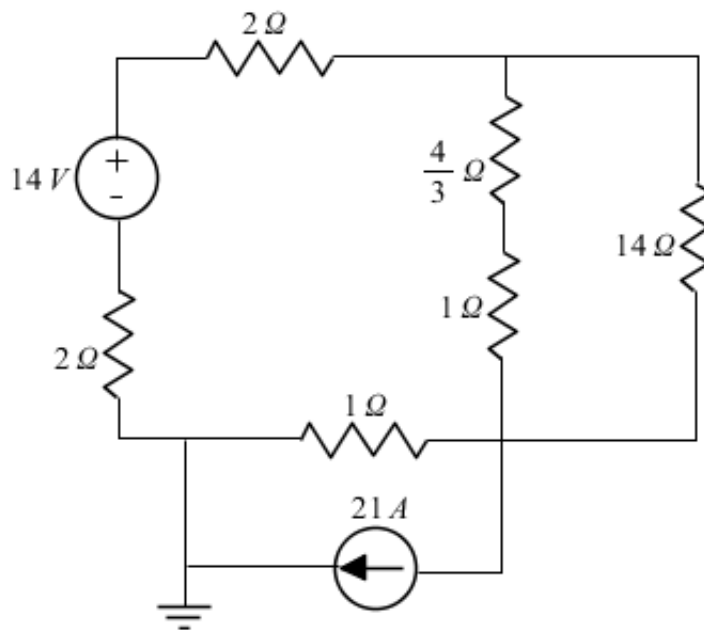
Figure 6: Equivalent circuit - Combined parallel resistors

Combine the series resistors and then the parallel resistors on the right branch.

$$R_{eq3} := \left(\frac{1}{(R_{eq1} + R4)} + \frac{1}{R5} \right)^{-1} = 2$$

The new circuit is
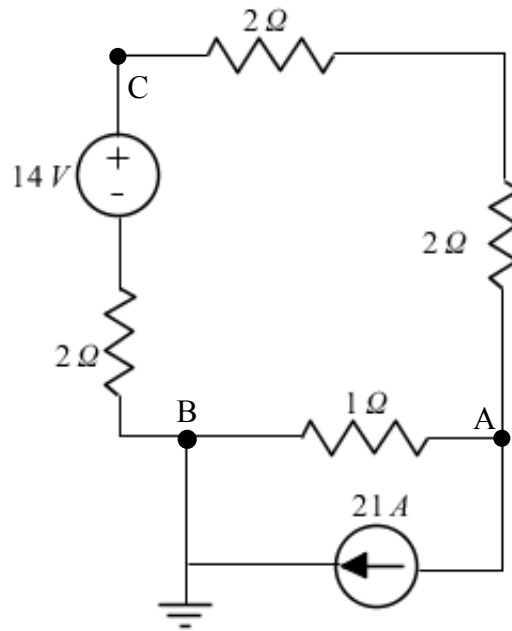
Figure 7: Equivalent circuit - Combining series and parallel circuits

Labelled nodes are on the previous diagram. The set of two 2 ohm resistors will combine to 4 ohms because they are in series. Apply KCL at node A.

$$eq1 := \frac{(v_c - v_a)}{2 + 2} = \frac{(v_a - v_b)}{1} + 21 :$$

Node B is connected directly to the ground, so it has a voltage of 0.

$$v_b := 0 :$$

Analyze the currents entering and exiting node B to find another equation with v_c.

$$eq2 := \frac{(v_a - v_b)}{1} + 21 = \frac{(v_b - (v_c - 14))}{2} :$$

Substitute v_b into the equations. Solve the first equation for v_c.

$$v_c := 5 \cdot v_a + 84 :$$

Put this into the second equation to solve for v_a, and subsequently v_c.

$$eq2 = v_a + 21 = -35 - \frac{5}{2} v_a \xrightarrow{\text{solve for v\_a}} [[v_a = -16]]$$
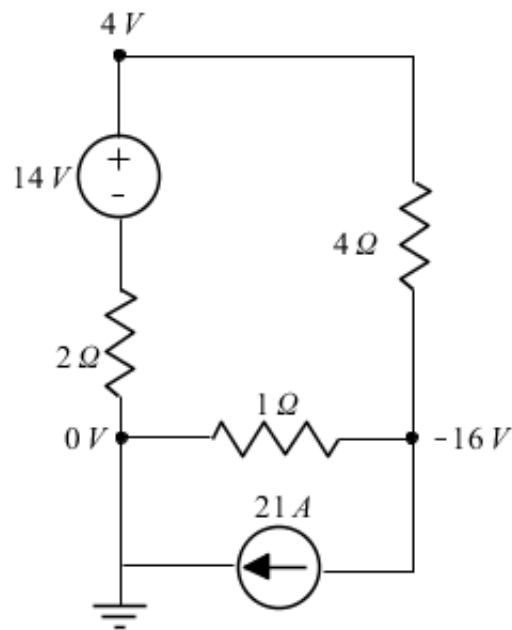
$$v_a := -16 :$$

$$v_c = 4$$

Figure 8: Combined series resistors

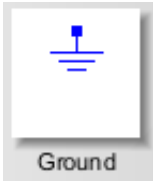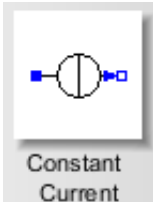The current across the resistor can be found with ohm's law.
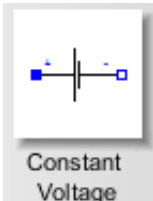
$$i = \frac{(-16 - 0)}{1} = i = -16$$

Therefore, the current flows from the ground to node A at 16 amps.

MapleSim Solution

Step 1: Insert Components

Insert the following components into a new MapleSim workspace.

Component	Location
Resistor (8 required)	Electrical > Analog > Common
Ground	Electrical > Analog > Common
Constant Voltage	Electrical > Analog > Sources > Voltage
Constant Current	Electrical > Analog > Sources > Current

Step 2: Connect Components

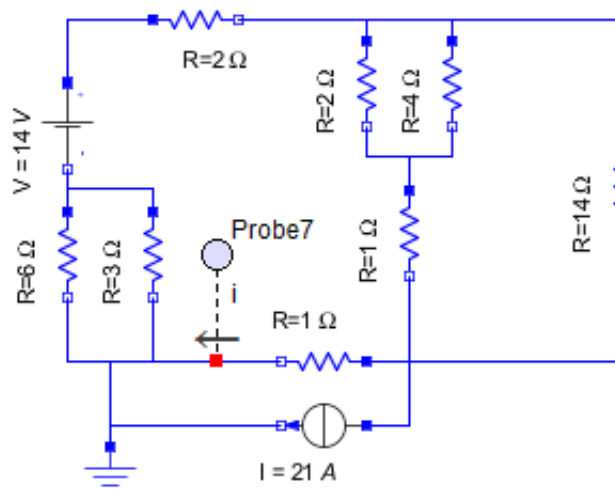Connect the components as shown in the following diagram.

Figure 9: MapleSim model diagram

Step 3: Set Up Parameters

1. Select a **resistor** block. On the 'Inpector' tab, set the *R* resistance parameter to the appropriate value as stated in the problem statement.
2. Repeat step 1 for the remaining resistors.
3. Select the **Constant Voltage** block. On the 'Inspector' tab, set the *V* voltage parameter to **14 V**.
4. Select the **Constant Current** block. Set the *I* current to **21 A**.
5. Attach a probe to the line from the bottom resistor. On the 'Inspector' tab, deselect the 'Voltage' box and select the 'Current' box to measure the current through the wire. Note that a probe will only measure voltage if it is attached to the flange, not the wiring, of an electrical element.

Step 4: Run Simulation

Run the simulation. A graph will be generated displaying the current through the resistor. This will be consistent with the analytical solution.

▼ **Results**

This graph will be generated measuring the current at the specified point in the
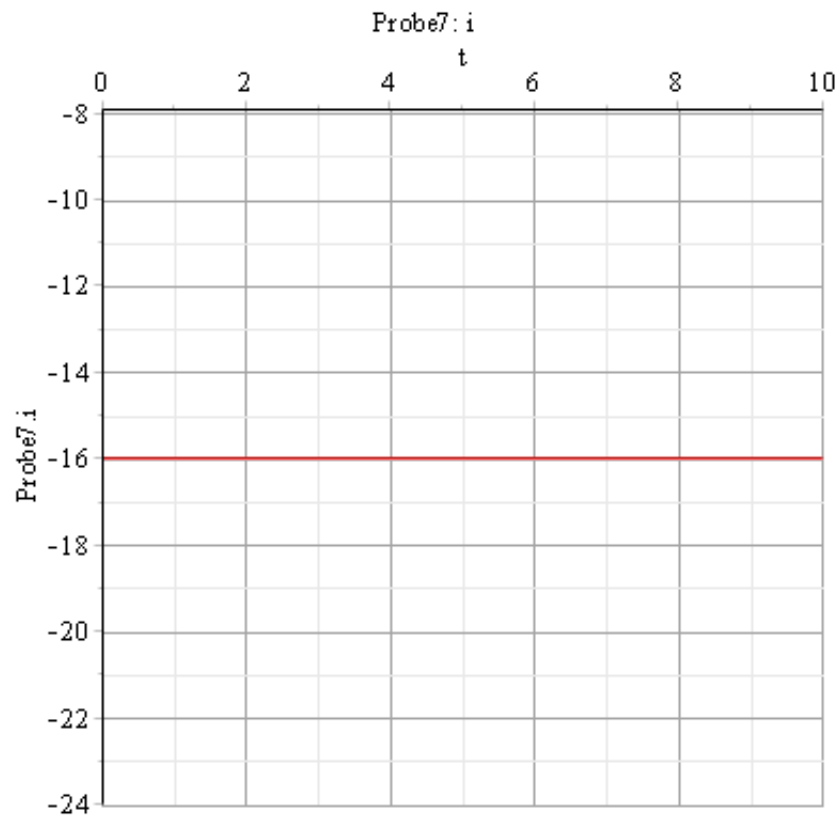
circuit.

Figure 10: Example 1 results

Example 2: Summing Amplifier

Problem Statement: The circuit below is a summing amplifier. The operational amplifier will output a voltage and current that will negate the current at the input and equate the voltage of the two inputs. Find the voltage V across the 4 ohm resistor at the op amp.
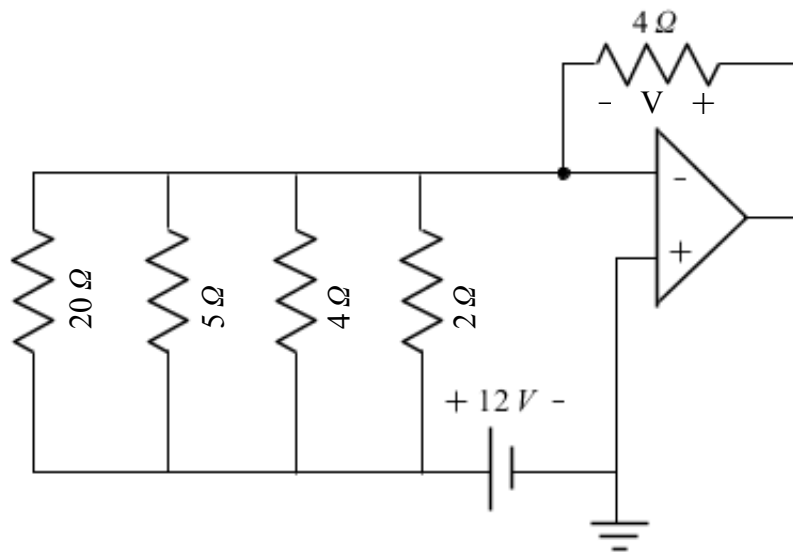
Figure 11: Summing amplifier circuit diagram

▼ Analytical Solution

restart;

Data:

$R1 := 20$:
$R2 := 5$:
$R3 := 4$:
$R4 := 2$:
$V_s := 12$:
$R_0 := 4$:

Solution:

Simplify the circuit and find the equivalent resistance. Four resistors in series can be combined using eq. (12). The equivalent resistance is,

$$R_{eq} := \left(\frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3} + \frac{1}{R4} \right)^{-1} = 1$$

The voltage difference across the parallel resistors is 12 V because the op amp inputs will have an equal voltage and the other input is connected to the ground. This make a $(12-0)$ V voltage difference over the resistors.

Using ohm's law, the equivalent current through the parallel resistors will be

$$i_{eq} := \frac{V_s - 0}{R_{eq}} = 12$$

This is equivalent to the current absorved by the op amp. Therefore, the current through the 4 ohm resistor R_0 is 12 amps. The voltage at the negative terminal of the op amp is zero, and the voltage across the resistor can be found using ohm's law.
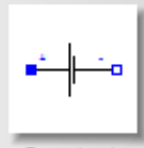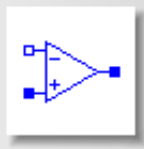
$$V = i_{eq} \cdot R_0 = V = 48$$

Therefore, the voltage difference across the resistor will be 48 volts. The current flows towards the op amp output, therefore the op amp must output -48 volts.

MapleSim Solution

Step 1: Insert Components

Drag the following components into a new workspace.

Component	Location
■-\/\/\/-□ Resistor (5 required)	Electrical > Analog > Common
	Electrical > Analog > Common

Ground	
Constant Voltage	Electical > Analog > Sources > Voltage
Ideal Op Amp 3Pin	Electrical > Analog > Operational Amplifiers
Voltage Sensor	Electrical > Analog > Sensors

Step 2: Connect the components.

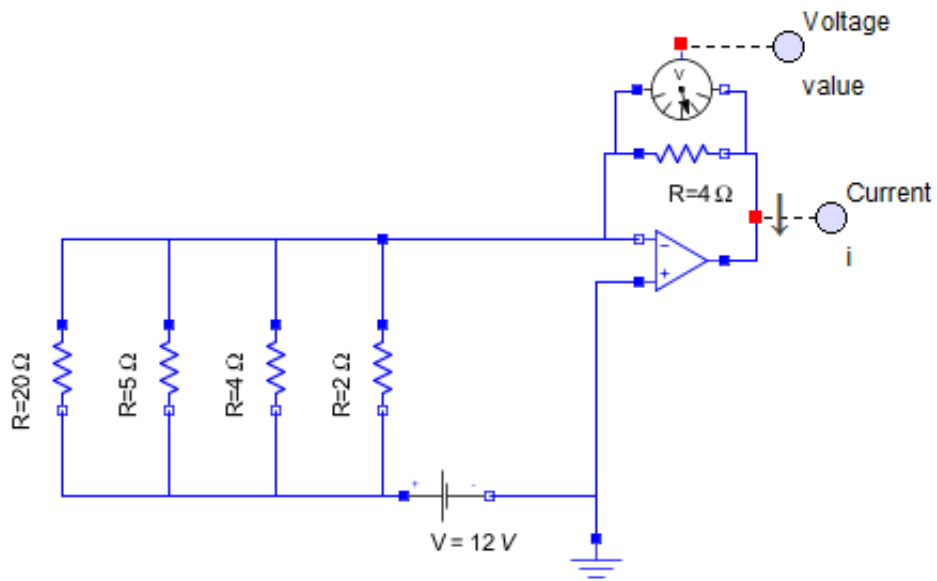Connect the components as shown in the diagram below.

Figure 12: MapleSim diagram of a summing amplifier

Step 3: Set Up Variables

1. Select a **Resistor** block. On the 'Inspector' tab, set the R parameter to **20** ohms.
2. Repeat step 1 for the remaining resistors of **5**, **4**, **4**, and **2** ohms.
3. Select the **Constant Voltage** block. Set the voltage V to **12** volts.
4. Connect a **Probe** to the output of the voltage sensor. Connect another probe

to the op amp/resistor connection, and on the 'Inspector' tab select the **Current** box and deselect the **Voltage** box.

Step 3: Run Simulation

Run the simulation. The generated plots will display the voltage and current through the resistor.

▼ **Results**

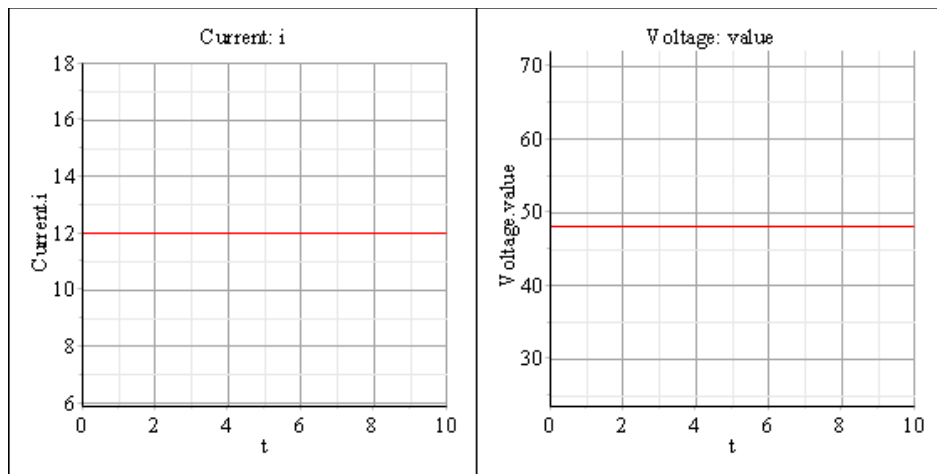The probes will generate the following graphs in the simulation.

Figure 13: Example 2 results

References:

R. Dorf, J. Svoboda. "Introduction to Electric Circuits", 8th Edition. RRD Jefferson City, 2010, John Wiley and Sons, Inc.